Super-resolution Sparse Channel Estimation for Localization - Prospects and Issues

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Joint Work with TU Graz

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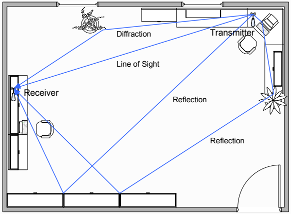
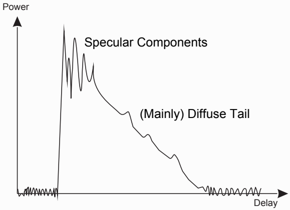
JS18 URSI France

Context / Motivation I

Multipath propagation:



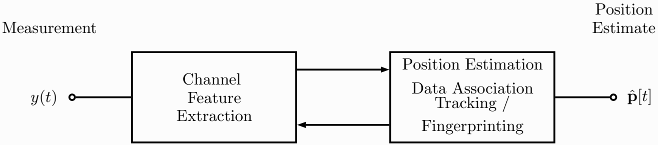


Localization exploiting multipath propagation:

- Multipath-propagarion-based finger printing
- Multipath-propagation-based simultaneous localization and mapping (SLAM)

Context / Motivation II

Generic architecture of a (two-stage) localization algorithm:



Relevant issues regarding the channel estimator:

- Detection of artefacts,
- Miss of components
- Superresolution capability
- Physical reality of estimated multipath components

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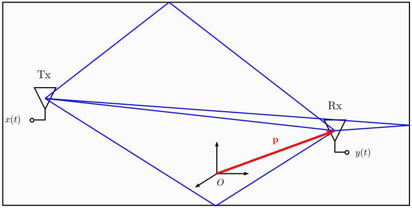
The channel as a linear time-invariant system:



Frequency domain characterization:

- $X(f)$: transmitted signal with bandwidth B
- $Y(f) \triangleq Y(f;\boldsymbol{p})$: received signal
- $W(f)$: White Gaussian noise
- $H(f) \triangleq H(f;\boldsymbol{p})$: Channel (frequency) transfer function

Input-output relationship:

$$Y(f) = H(f)X(f) + W(f)$$

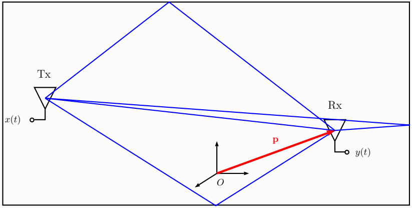
Traditional assumption:

$$H(f) = \sum_{\tilde{\ell}}^{\tilde{L}} \tilde{\alpha}_{\tilde{\ell}} \exp\{-j2\pi f \tilde{\tau}_{\tilde{\ell}}\}$$

$\tilde{L}$ small: specular channel; $\tilde{L}$ large: dense (diffuse) channel

Signal Model - Time Dispersion

The channel as a linear time-invariant system:



Channel sounding (OFDM):

- $x(t)$: transmitted signal
- $y(t) \triangleq y(t;\boldsymbol{p})$: received signal
- $w(t)$: white Gaussian noise
- $H(f) \triangleq H(f;\boldsymbol{p})$: channel (frequency) transfer function

Traditional assumption:

$$H(f) = \sum_{\tilde{\ell}}^{\tilde{L}} \tilde{\alpha}_{\tilde{\ell}} \exp\{-j2\pi f \tilde{\tau}_{\tilde{\ell}}\}$$

$\tilde{L}$ small: specular channel; $\tilde{L}$ large: dense (diffuse) channel

Discrete-time Model

$$\boldsymbol{y} = \boldsymbol{h} + \boldsymbol{w}$$

where

- $\boldsymbol{y} \in \mathbb{C}^N$: received symbol
- $\boldsymbol{w} \in \mathbb{C}^N$: channel noise
- We assume an all-one OFDM symbol $[1, \ldots, 1]^\mathsf{T}$.

and

$$\boldsymbol{h} \triangleq [h_1, \ldots, h_N]^\mathsf{T} \triangleq [H(n\Delta f) : n = 0, \ldots, N-1]^\mathsf{T}$$

with

- Δf: carrier spacing
- N: number of carrier

Under the traditional assumption:

$$\boldsymbol{h} = \boldsymbol{\Psi}(\tilde{\boldsymbol{\theta}})\, \tilde{\boldsymbol{\alpha}}$$

$$\boldsymbol{\Psi}(\tilde{\boldsymbol{\theta}}) \triangleq [\psi(\tilde{\theta}_{\tilde{\ell}}) : \tilde{\ell} = 1, \ldots, \tilde{L}]$$

$$\psi(\theta) \triangleq [\exp\{-j2\pi n\theta\} : n = 0, \ldots, N-1]^\mathsf{T}$$

Frequency variable:
$\theta \in [-1/2, +1/2]$

$$\tilde{\theta}_{\tilde{\ell}} \triangleq \Delta f \tilde{\tau}_{\tilde{\ell}}$$

First- and Second-order Characterization

Expectation:

$$\mathbb{E}[\boldsymbol{h}] = \mathbb{E}[\boldsymbol{w}] = \mathbb{E}[\boldsymbol{y}] = 0$$

Covariance matrices:

$$\begin{aligned} \boldsymbol{\Sigma}_{\boldsymbol{hh}} &\triangleq \mathbb{E}[\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}] \qquad \text{Channel covariance matrix} \\ \boldsymbol{\Sigma}_{\boldsymbol{ww}} &= \beta^{-1}\boldsymbol{I} \\ \boldsymbol{\Sigma}_{\boldsymbol{yy}} &= \boldsymbol{\Sigma}_{\boldsymbol{hh}} + \beta^{-1}\boldsymbol{I} \end{aligned}$$

Randon vectors are assumed circularly symmetric.

Rank of the channel covariance matrix:

$$L \triangleq \operatorname{rank}(\boldsymbol{\Sigma}_{\boldsymbol{hh}}) \leq N$$

The rank is system-dependent: it depends on N for fixed Δf.

Carathéodory Representation

If $\boldsymbol{\Sigma}_{\boldsymbol{hh}}$ is Toeplitz, it can be decomposed as

$$\begin{aligned}\boldsymbol{\Sigma}_{\boldsymbol{hh}} &= \sum_{\ell=1}^{L} \gamma_\ell \boldsymbol{\psi}(\theta_\ell)\boldsymbol{\psi}(\theta_\ell)^{\mathsf{H}} \\ &= \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\Gamma}\boldsymbol{\Psi}(\boldsymbol{\theta})^{\mathsf{H}}\end{aligned}$$

$$\begin{aligned}&L = \mathrm{rank}(\boldsymbol{\Sigma}_{\boldsymbol{hh}}) \in \{1,\ldots,N\} \\ &\boldsymbol{\gamma} \triangleq [\gamma_\ell : \ell = 1,\ldots,L]^{\mathsf{T}} \in (0,\infty)^L \\ &\boldsymbol{\theta} \triangleq [\theta_\ell : \ell = 1,\ldots,L]^{\mathsf{T}} \in [-1/2,+1/2)^L \\ &\boldsymbol{\Gamma} \triangleq \mathrm{diag}(\boldsymbol{\gamma})\end{aligned}$$

If $L < N$, the representation is unique.

It follows that

$$\boldsymbol{h} = \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\alpha} \qquad \boldsymbol{\alpha} \triangleq [\alpha_\ell : \ell = 1,\ldots,L]^{\mathsf{T}} \in \mathbb{C}^L$$

Under the reasonable assumption that $\boldsymbol{h}$ is wide-sense stationary, $\boldsymbol{\Sigma}_{\boldsymbol{hh}}$ is Toeplitz.

We assume that $L < N$, see numerical results.

Line Spectral Estimation

Generic model:

$$\boldsymbol{y} = \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\alpha} + \boldsymbol{w} = \sum_{\ell=1}^{L} \alpha_\ell \boldsymbol{\psi}(\theta_\ell) + \boldsymbol{w}$$

Inference problem:

Estimate L, $(\alpha_\ell, \theta_\ell); \ell = 1, \ldots, L$, and noise precision β!

The fact that L is unknown makes the problem more difficult ... and scientifically challenging.

Back to the traditional assumption:

Traditional assumption leads to the same representation of $\boldsymbol{h}$:

$$\boldsymbol{h} = \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\alpha} = \boldsymbol{\Psi}(\tilde{\boldsymbol{\theta}})\tilde{\boldsymbol{\alpha}}$$

While the components in $\boldsymbol{\Psi}(\tilde{\boldsymbol{\theta}})\tilde{\boldsymbol{\alpha}}$ are intended to have a physical meaning, those in $\boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\alpha}$ do not. The latter are virtual components. They might coincides with a phisical component, but not always.

A sparse channel estimator exploits the low-rank structure of $\boldsymbol{\Sigma}_{\boldsymbol{hh}}$. Thus it estimates the virtual components.

Signal Model for Inference

We fix the number of components:

$$\boldsymbol{y} = \sum_{m=1}^{M} \alpha_m \psi(\theta_m) + \boldsymbol{\alpha} \qquad M \geq N > L$$

$$= \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\alpha} + \boldsymbol{\alpha}$$

$\boldsymbol{\Psi}(\boldsymbol{\theta}) = [\psi(\theta_1), \ldots, \psi(\theta_M)]$ Dictionary matrix (frequency dependent)

Rationale:

We use a sparse estimator that will set the estimates of the weight of surnumerous components to zero.

Our Choice: Sparse Bayesian learning

We use a Bayesian framework.

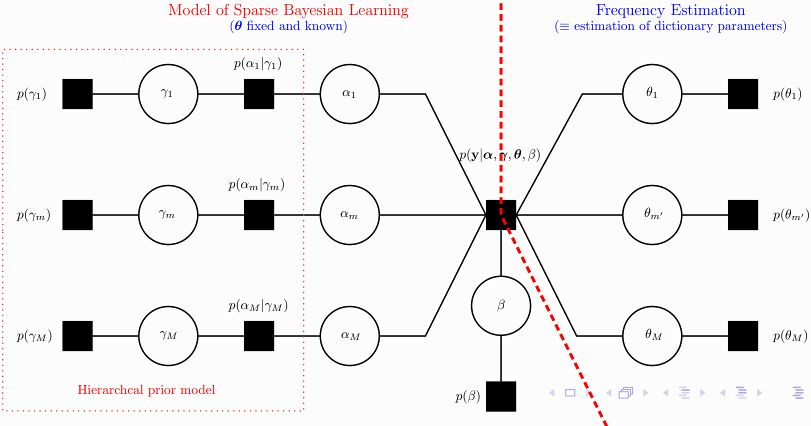
We use a sparsity inducing hierarchical prior for $\boldsymbol{\alpha}$, specifically a Gamma-Gaussian prior.

Sparse Bayesian Learning with Parametric Dictionary Estimation

Joint probability density function (pdf):

$$p(\boldsymbol{y},\boldsymbol{\alpha},\boldsymbol{\gamma},\boldsymbol{\theta},\beta) = \underbrace{p(\boldsymbol{y}|\boldsymbol{\alpha},\boldsymbol{\theta},\beta)}_{p(\boldsymbol{y}|\boldsymbol{\alpha},\boldsymbol{\Psi}(\boldsymbol{\theta}),\beta)} \underbrace{\prod_{m=1}^{M} p(\alpha_m|\gamma_m) \prod_{m'=1}^{M} p(\gamma_{m'})}_{\text{Hierarchical prior model}} \, p(\beta) \prod_{m''=1}^{M} p(\theta_{m''})$$

Factor graph:



Probabilistic Model in the Sparse Bayesian Framework

Joint pdf:

$$p(\boldsymbol{y}, \boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\theta}, \beta) = p(\boldsymbol{y}|\boldsymbol{\alpha}, \boldsymbol{\theta}, \beta) \prod_{m=1}^{M} p(\alpha_m|\gamma_m) \prod_{m'=1}^{M} p(\gamma_{m'}) \prod_{m''=1}^{M} p(\theta_{m''})\, p(\beta)$$

where

$$p(\boldsymbol{y}|\boldsymbol{\alpha}, \boldsymbol{\theta}, \beta) = p_{\mathrm{CN}}(\boldsymbol{y}; \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\alpha}, \beta^{-1}\mathbf{I})$$

i.e. $\mathbf{n}$ is a white Gaussian noise vector

$$p(\alpha_m|\gamma_m) = p_{\mathrm{CN}}(\alpha_m; 0, \gamma_m^{-1}), \quad m = 1, \dots, M$$

$$p(\gamma_m) = p_{\mathrm{G}}(\gamma; c, d), \quad m = 1, \dots, M$$

$$p(\theta_m) = p_{\mathrm{VM}}(\theta_m; \mu, \kappa), \quad m = 1, \dots, M$$

$$p(\beta) = p_{\mathrm{G}}(\beta; a, b)$$

The distribution we will consider subsequently, specifically their pdfs:

- $p_{\mathrm{CN}}(\mathbf{x}; \mathbf{m}, \mathbf{V}) = \frac{1}{\pi^M |\mathbf{V}|} \exp\{-(\mathbf{x} - \mathbf{m})^{\mathrm{H}} \mathbf{V}^{-1} (\mathbf{x} - \mathbf{m})\}$ complex Gaussian
- $p_{\mathrm{VM}}(\theta; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} \exp\{\kappa \cos(\theta - \mu)\}, \quad \theta \in [-\pi, +\pi)$ von Mises
- $p_{\mathrm{G}}(x; u, v) = \frac{b^a}{\Gamma(a)} x^{-u-1} \exp\{-v/x\}, \quad x > 0$ Gamma

Mean-Field Approximation

Posterior pdf:

$$p(\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\theta}, \beta | \mathbf{y}) \propto p(\mathbf{y} | \boldsymbol{\alpha}, \boldsymbol{\theta}, \beta) \prod_m p(\alpha_m | \gamma_m) \prod_{m'} p(\gamma_{m'}) \prod_{m''} p(\theta_{m''})\, p(\beta)$$

Family of proxy pdfs:

We consider a family $\mathcal{Q}$ of pdfs with the "simpler" factorization

$$q(\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\theta}, \beta) = q(\boldsymbol{\alpha}) \prod_m q(\gamma_m) \prod_{m'} q(\theta_{m'})\, q(\beta)$$

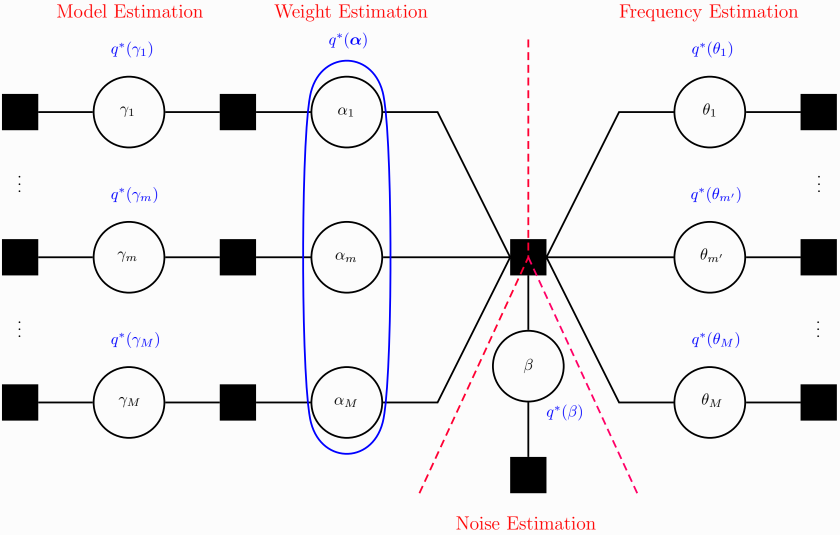
We select as an approximation of $p(\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\theta}, \beta | \mathbf{y})$ the one element in $\mathcal{Q}$ that is the closest:

$$q^*(\mathbf{z}) = \underset{q(\mathbf{z}) \in \mathcal{Q}}{\arg\min}\, \mathrm{KL}(q(\mathbf{z}) \| p(\mathbf{z} | \mathbf{y})) \qquad \mathbf{z} \triangleq (\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\theta}, \beta).$$

Computation of $q^*(\mathbf{z})$:

Different iterative approaches exist to compute (approximate) the solution to the arg min problem (e.g. variational EM).

Mean-Field Approximation - The Four Estimation Tasks



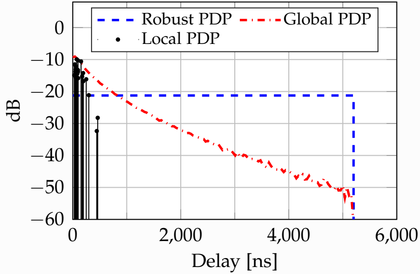
Facts about and Properties of SBL Estimators

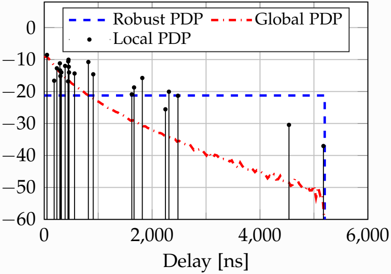
- Various hierarchical models for α can be used that promote sparsity: Gamma-Gaussian, Bernouilli-Gaussian or Bernouilli-Gamma-Gaussian.
- SBL are Type II Bayesian estimators, while conventional methods, such as basis pursuit denoising (LASSO), atomic norm denoising, can be interpreted as Type I Bayesian estimator.
- Type II Bayesian estimators promote greater sparsity than Type I Bayesian estimators.
- SBL can also be interpreted as stochastic maximum-likelihood with model order estimation.
- SBL inherently integrates the order estimation process. No hypothesis testing problem needs to be solved, like in classical model order estimation based on information theoretic criteria.

Numerical Studies - Rank of Radio Channels

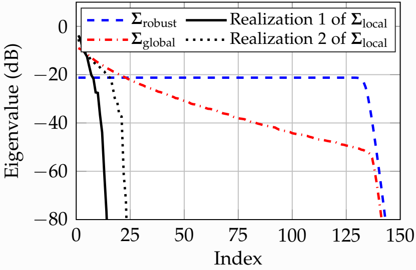
A: Specular Channel - ITU-R M.2135 UMa NLOS Channel Model

Two generated impulse responses:





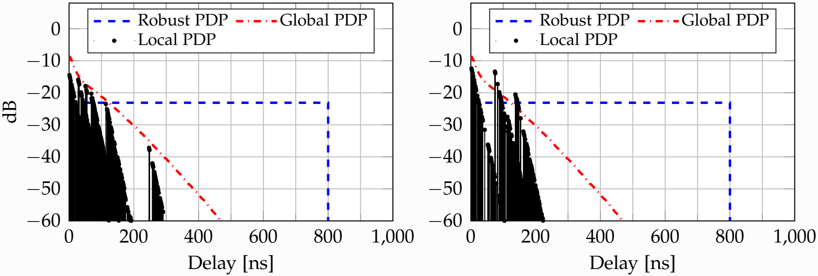
Covariance eigenvalues of the two responses:



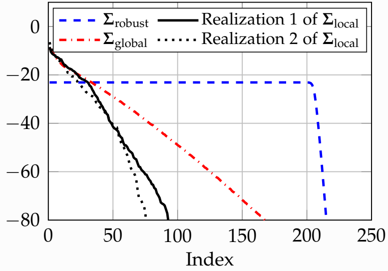
- Bandwidth: 25.6 MHz
- Sampling period: 25 KHz
- Dimension of $\boldsymbol{y}$: 1024

B: Dense Channel - IEEE 802.15.a Outdoor NLOS Channel Model

Two generated impulse responses:

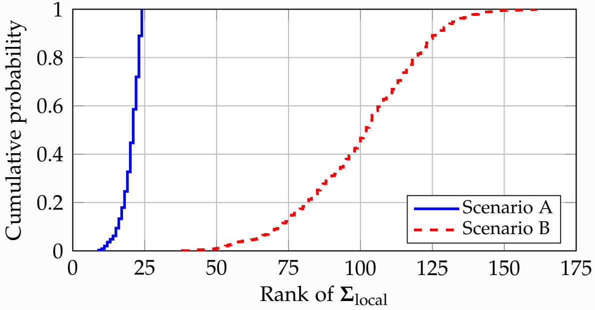


Covariance eigenvalues of the two responses:



- Bandwidth: 256 MHz
- Sampling period: 250 KHz
- Dimension of $\boldsymbol{y}$: 1024

Effective Rank of the Synthetic Channels



Experimental Results

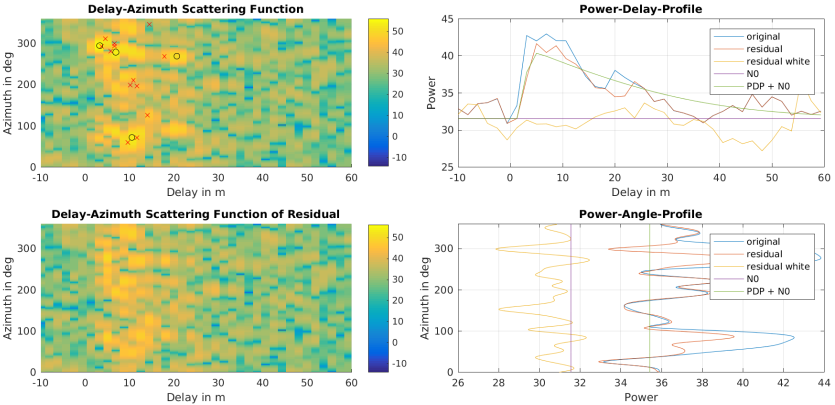
Description of the Measurement Experiment

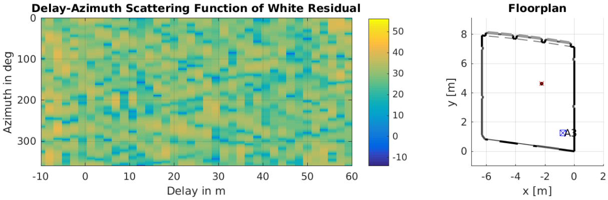
Investigated indoor environment:

Equipment:

- Center frequency: 7 GHz
- Agents equipped with a 5x5 (virtual) array; array spacing: 2 cm
- Anchors equipped with a single antenna
- Bandwidth is varied: 100, 200, 500, 1000 MHz

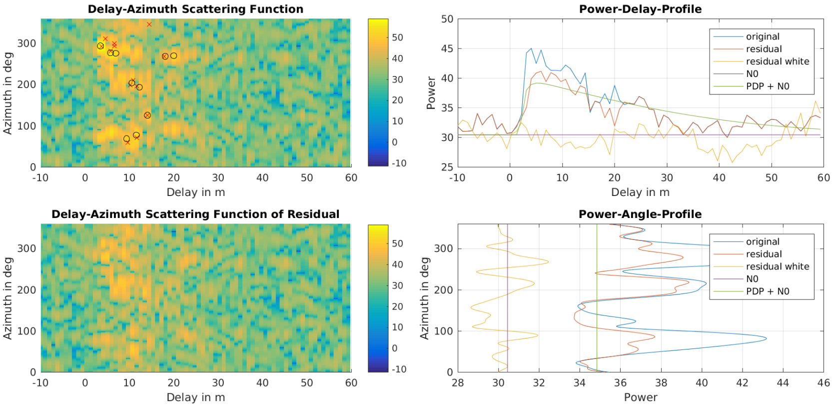
Bandwidth=100MHz, Agent 3, Anchor 3

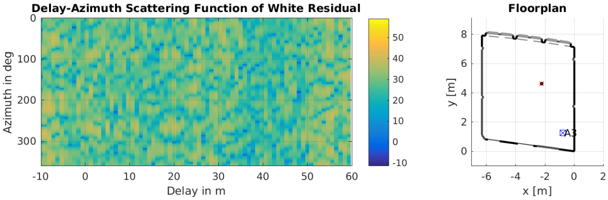




Bandwidth: 100 MHz
Agent 3 & Anchor 3
Found 4/13 specular components

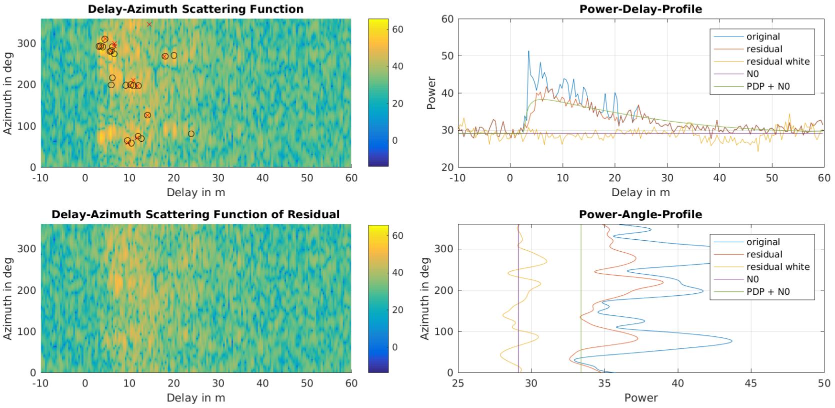
Bandwidth=200MHz, Agent 3, Anchor 3

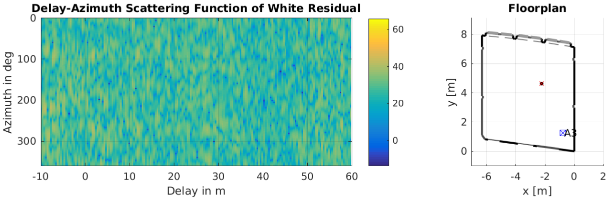




Bandwidth: 200 MHz
Agent 3 & Anchor 3
Found 10/13 specular components

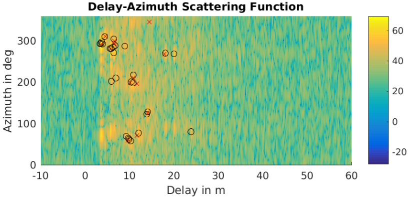
Bandwidth=500MHz, Agent 3, Anchor 3

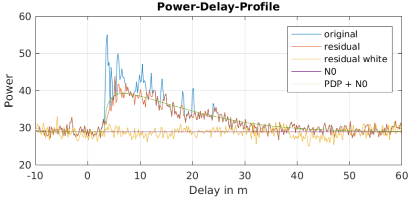


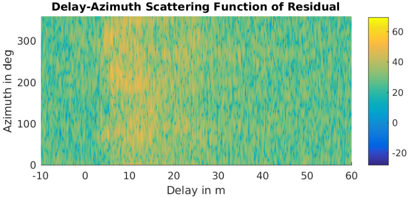


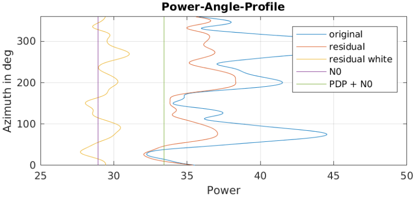
Bandwidth: 500 MHz
Agent 3 & Anchor 3
Found 22/13 specular components

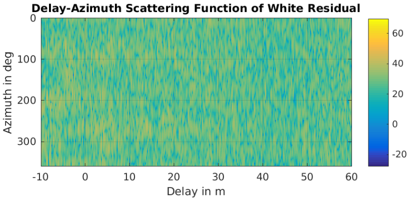
Bandwidth=1000MHz, Agent 3, Anchor 3

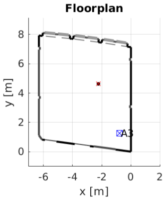






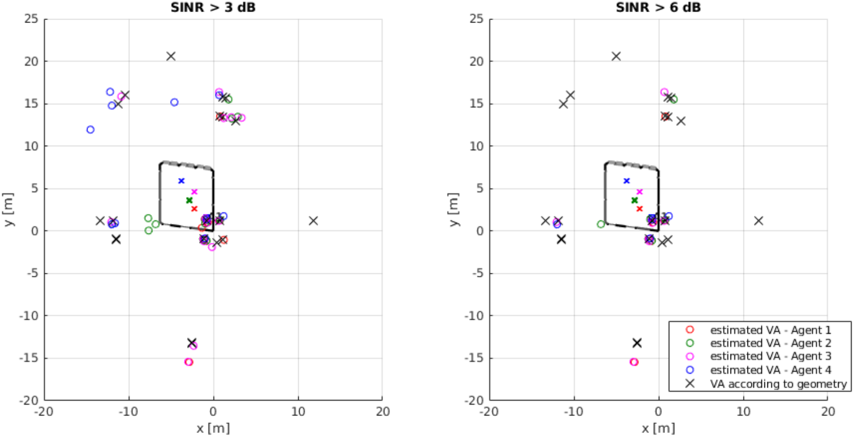






Bandwidth: 1000 MHz
Agent 3 & Anchor 3
Found 27/13 specular components

Reconstructed and Estimated Mirror Sources



Conclusion and Outlook

Conclusion:

- The inherent threshold of SBL needs to be increased in order to decrease the number of artefacts (VALSE is an alternative).
- Superresolution demonstrated in synthetic channels (half the Nyquist period)
- SBL shows a sensible behaviour in real conditions.
- Caution is needed in the interpretation of estimated components as physical ones.
- Behaviour is strongly dependent on the selected iterative implementation.
- SBL works for both specular-like and diffuse channels

Outlook:

- Implementation in a localization estimation and tracking system

References

[BHF17] M. A. Badiu, T. Hansen, and B. Fleury, Variational bayesian inference of line spectra, IEEE Transactions on Signal Processing **PP** (2017), no. 99, 1–1.

[HFR18] T. L. Hansen, B. H. Fleury, and B. D. Rao, Superfast line spectral estimation, IEEE Transactions on Signal Processing **PP** (2018), no. 99, 1–1.