iltrage de Kalman invariant pour la navigation inertielle hybridée
Invariant Kalman filtering for GPS-aided inertial navigation

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Résumé / Abstract

Le filtre de Kalman, ou plus précisément filtre de Kalman étendu (EKF), est un outil fondamental de l'ingénieur très utilisé dans le domaine de la navigation. Le domaine récent du filtrage de Kalman dit "invariant", est consacré à l'utilisation de la géométrie sous-jacente de l'espace d'état et de la dynamique pour modifier et améliorer l'EKF classique, notamment en termes de garanties de convergence. Les principales applications de cette méthodologie sont la localisation, la navigation, mais aussi la localisation et cartographie simultanée, connue sous le nom de SLAM en anglais, pour lequel l'EKF invariant résout les problèmes bien connus d'incohérence de l'EKF classique. Bien que la méthodologie soit récente, les remarquables propriétés du filtre de Kalman invariant ont d'ores et déjà motivé une implémentation industrielle dans le domaine de la navigation, par Safran Electronics & Defense, anciennement Sagem. Le but de cet exposé est de proposer une introduction au filtrage de Kalman invariant, et de fournir les grandes intuitions permettant de comprendre sa supériorité par rapport au filtrage de Kalman étendu classique en ce qui concerne le domaine de la navigation haute précision.

Invariant Kalman filtering for GPS-aided inertial navigation

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Joint work with A. Barrau (Safran Tech / SAGEM)

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Introduction

The extended Kalman filter (EKF) has been the most widespread tool for sensor fusion since the 1960s for navigation.

Developped by NASA in the 1960s in the Apollo program.

SAGEM: pioneered industrial applications to navigation, notably thanks to Pierre Faurre.

Rudi Kalman

Pierre Faurre

Introduction

The invariant extended Kalman filter (IEKF) introduced a decade ago is a modification to EKF based on symmetries of the problem.[1]

Safran Electronics & Defense (SAGEM) chose to invest through the PhD of A. Barrau (2012-2015). The company has registered three patents for applications of invariant filtering.

This talk: some industrial results, and some insights from a simplified example.

[1] For an overview see "Invariant Kalman Filtering". Barrau & Bonnabel. The Annual Reviews. 2018

Industrial results

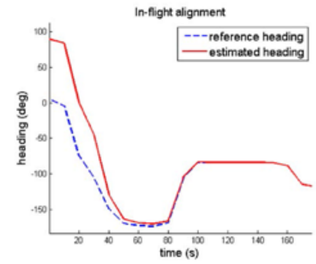
Safran Electronics & Defense (SAGEM) has released an industrial product using the Invariant extended Kalman filter, the Euroflir 410, embedded in the drone Patroller.

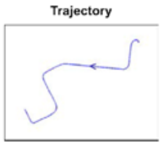
Euroflir 410

Patroller drone

Industrial results

In flight alignment consists in finding the orientation of the vehicle from inertial sensors and GPS.





Industrial results

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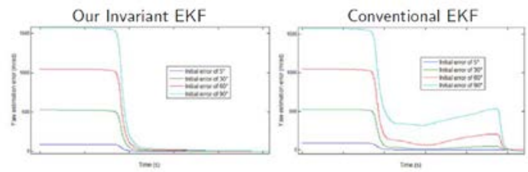


Estimation error, with initial error from 5° to 90°

Industrial results

Why is alignment challenging for the EKF ?

- Highly accurate inertial sensors
- Vertical easy to find (≈ known)
- Heading unknown⇒ possible large estimation error

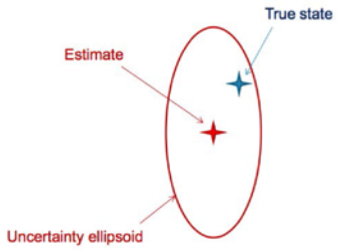
Caveats of the EKF ?

- Designed for small errors
- Here the estimate lives a (constrained) subspace of the state space.

Remainder of the talk devoted to provide insight on the superiority of the IEKF for this type of problems.

Preliminary

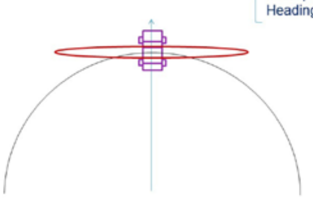
Graphical representation: The EKF computes an estimate $\hat{X}_t$ with a covariance matrix P_t. This is the confidence ellipsoid.



A simple terrestrial navigation problem

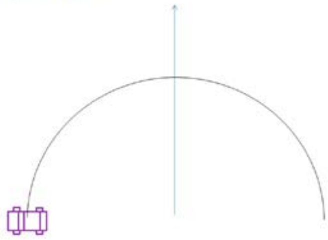
Introduction : Extended Kalman Filter

- Accurate odometer
- Initial position known
- Heading unknown



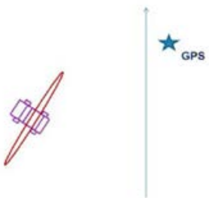
A simple terrestrial navigation problem

Non-linear system : estimate



A simple terrestrial navigation problem

This is what the EKF **should** do



Reminder: EKF equations

Consider continuous time deterministic dynamics with discrete time measurements $t_0, t_1, \cdots$

$$\frac{d}{dt}X_t = f(X_t, U_t), \quad Y_n = h(X_t) + V_n$$

The EKF is based on a prior distribution with $\hat{X}_0,\ P_0$.

Propagation step: for $t_{n-1} < t < t_n$

$$\frac{d}{dt}\hat{X}_t = f(\hat{X}_t, U_t) \quad \frac{d}{dt}P_t = A_t P_t + P_t A_t^T$$

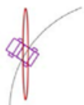
Update step: at $t = t_n$,

$$K_n = P_{t_n} H_n^T \left(H_n P_{t_n} H_n^T + N\right)^{-1}$$
$$\hat{X}_{t_n}^+ = \hat{X}_{t_n} + K_n(Y_n - \hat{X}_{t_n})$$
$$P_{t_n}^+ = (I - K_n H_n) P_{t_n}.$$

A simple terrestrial navigation problem

And the EKF **does this**

A simple terrestrial navigation problem



Obviously the EKF poorly handles small eigenvalues in P_t.

Use of symmetries as the remedy.

Namely: we use an Invariant (I)-EKF [2].

[2] See "Invariant Kalman filtering", Annual Reviews, 2018.

Invariant Kalman filtering

Main difference the estimation error it linearizes is not

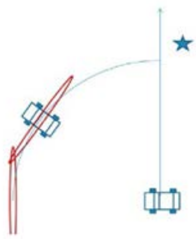
$$\begin{pmatrix} \hat{\theta} - \theta \\ \hat{x}_t - x_t \end{pmatrix}, \quad \text{where} \quad \theta \in S^1,\ x \in \mathbb{R}^2$$

It is the estimation error

$$\begin{pmatrix} \hat{\theta} - \theta \\ R(\theta)^T (\hat{x}_t - x_t) \end{pmatrix}$$

where $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

Invariant Kalman filtering



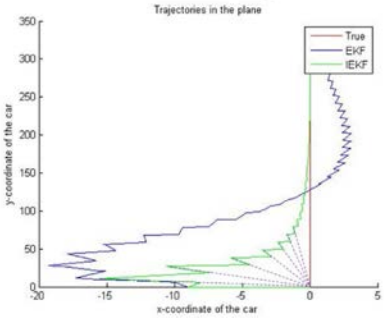
Estimation error

$$\begin{pmatrix} \hat{\theta} - \theta \\ R(\theta)^T (\hat{x}_t - x_t) \end{pmatrix}$$

Theorem: The estimate belongs only to sets "physically" reachable.

Invariant Kalman filtering

Illustration with simulations for a straight line motion.



Invariant Kalman filtering

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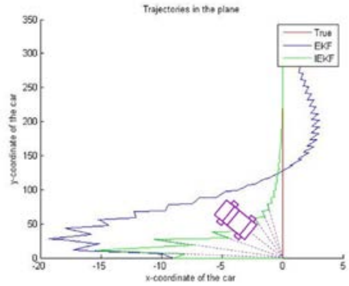

Invariant Kalman filtering

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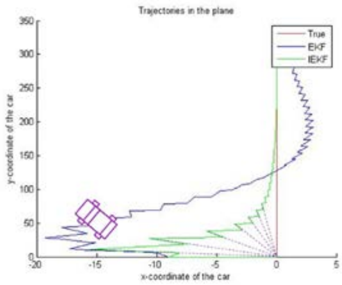

Invariant Kalman filtering

Convergence: what about it? We can prove it!

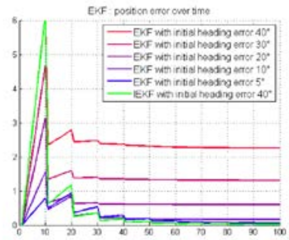

Figure: Position error of the EKF for GPS and perfect odometry, with initial position known and initial heading unknown. The IEKF initialized with a large error (40°) even beats the EKF initialized with a small error (5°).

Invariant Kalman filtering

Convergence: The EKF truly does NOT converge

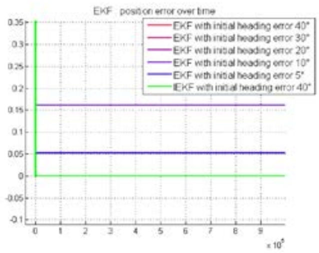

Figure: Extension to 1 million steps. The EKF does not converge.

Conclusion and perspectives

The Invariant consists of using alternative coordinates dictated by symmetries for EKF design.

Industrial application: the problem we considered serves as a simplified in flight alignment problem, an inevitable task for industrial high precision inertial navigation

Other applications (Signal Processing): Object tracking with radars (see next talk by Marion Pilté).

Other applications (Robotics): Visual inertial odometry (see work of Martin Brossard).