

Benefits of space-time diversity for radar *Les bienfaits de la diversité spatio-temporelle en radar*

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Abstract

When designing a new radar system, standard resolution trade-offs play a major role, providing the basic parameters of the radar, such as size, update rate, and range. Besides, diversity has long been used for mitigating fading effects due to the fluctuation of targets and clutter.

However, with the arrival of more flexible systems, using multiple parallel channels on transmit and receive, and wider instantaneous bandwidths, these standard trade-offs are becoming less simple – and more flexible. In this communication, we will analyze the benefits of diversity and its relations with range, Doppler, and angle, for detection and location of moving targets with wideband / wide beam radar systems. The idea is to contribute to a better understanding of the real benefits of agile transmissions for detection/localization of moving targets, focusing on range, velocity and angular measurement improvements, and on the benefits for detection of moving targets.

Special attention will be given to the quality of the different wideband wide-beam sensor modes for long range surveillance, and new results on detection of moving targets in clutter will be provided to demonstrate the effectiveness of these new architectures for small targets detection at long range, in difficult environments.

Résumé

Lors de la conception d'un nouveau système radar, les compromis classiques de résolution jouent un rôle majeur, pilotant les paramètres de base du radar, tels que la taille, la cadence de renouvellement et la portée. Par ailleurs, la diversité a longtemps été utilisée pour atténuer les effets de fading dus à la fluctuation des cibles et du fouillis.

Cependant, avec l'arrivée de systèmes plus souples, l'utilisation de plusieurs canaux parallèles en transmission et en réception, et des largeurs de bande instantanées plus larges, ces compromis standard deviennent moins simples - et plus flexibles. Dans cette communication, nous analyserons les apports de la diversité et ses relations avec la distance, le Doppler et l'angle, pour la détection et la localisation de cibles en mouvement avec des systèmes radar à large bande / large faisceau. L'idée est de contribuer à une meilleure compréhension des avantages réels des transmissions agiles pour la détection / localisation de cibles mobiles, en se concentrant sur les améliorations de portée, de vitesse et de mesure angulaire, et sur les avantages pour la détection des cibles en mouvement.

Une attention particulière sera accordée à la qualité des différents modes de détection pour la surveillance à longue portée, et de nouveaux résultats sur la détection des cibles mobiles en présence de fouillis seront présentés pour démontrer l'efficacité de ces nouvelles architectures pour la détection de petites cibles à longue portée, dans des environnements difficiles.

1 Introduction

1.1 Objectives

In order to design a surveillance radar¹, a critical point is the “illumination time”, also called the “time on target”: this time duration should be long enough to allow Doppler analysis, and to gain a sufficient signal to noise ratio (SNR), but also sufficiently small to allow a fast update rate, required by the user. This well-known trade-off between update rate and velocity resolution also involves the antenna beamwidth (the wider the beam, the better the velocity resolution, for a given update rate), and the clutter rejection capability (the wider the beam, the higher the clutter level), and has also direct consequences on the power budget (the wider the beam, the lower the antenna gain, but also the higher the coherent integration gain, for a given update rate).

These intricate relations between beamwidths, velocity resolution, and power budget (hence, range) are getting even more complex when taking into account the fluctuation characteristics of the targets and clutter, since

¹ This paper focuses on the basic issue of detection of moving targets with a ground surveillance radar, taking into account ground clutter.

performances can be improved through an increased averaging of clutter and target echoes – averaging which may itself be eased through widening of the beam, or longer illumination time. Such improvements are often more difficult to analyze, because they arise through modifications of the clutter and targets distribution functions, more complex than mere mean or standard deviation modifications (clutter and targets being generally not gaussian, averaging several samples generally changes the resulting distribution functions).

Moreover, modern radar systems generally operate over significant relative bandwidths – typically 10% – which can be exploited either coherently, with wider instantaneous bandwidths (providing improved range resolutions), or non-coherently, with a collection of measurements in different sub-bands. Again, these different operating modes have consequences on the power budget, but also on the distribution functions of targets and clutter.

The purpose of this paper is to try and clarify, with intuitive reasoning rather than precise equations, the order of magnitudes of these competing effects, so as to provide the designer with some basic insight necessary for building new radar architectures, involving multiple transmitting / receiving channels and arbitrary waveform agility.

1.2 Canonical problem

Detection being a 2-hypothesis problem (H_0 : no target, H_1 : a target), it basically comes down to comparing a certain quantity X , function of the received signals and of the expected situations (e.g. energy of the output of a matched filter), to a threshold depending on the required probability of detection P_d and probability of false alarm P_{fa} . This situation is shown on Figure 1, where the position of the threshold T defines the Probability of detection P_d (area with oblique lines) and the probability of false alarm P_{fa} (area with horizontal lines).

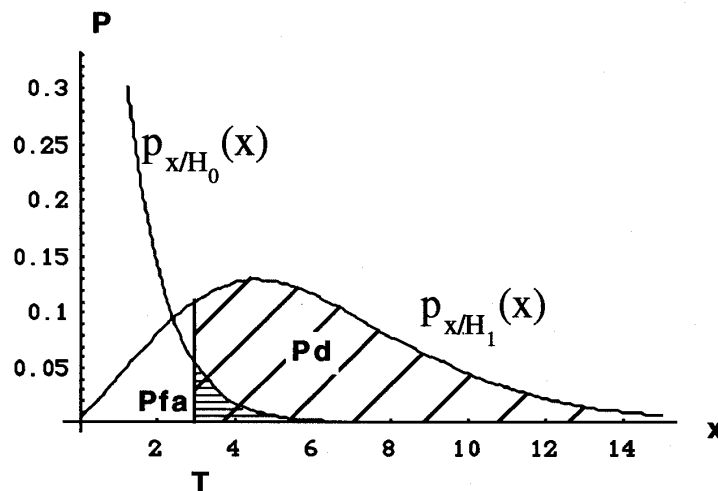


Figure 1: Detection and False alarm, after thresholding of the quantity X with threshold T (from [1]).

Obviously, the shape of the probability density functions p_{x/H_i} (probability of the received signal, under hypothesis H_i) is critical here. Using *diversity* is a means to improve the separation: generally speaking, averaging quantities is a way to reduce the spread of each probability density functions, and to bring it closer to a Gaussian (central limit theorem); Using *coherent integration*, or more generally matched filtering, is a way to increase the mean value of X under hypothesis H_1 . Both techniques thus improve the separation, in different ways: our objective here is to clarify these effects, and their consequences, for typical situations.

2 Standard Detection

Statistical Detection of radar fluctuating targets in the presence of noise is limited by the presence of noise and by the fact that the target may provide only very small signals for certain presentation angles or frequencies of illumination (a phenomenon also known as target fading in the literature). In order to mitigate target fading most radars use frequency agility:

- (1) They transmit successive bursts at different carrier frequencies;
- (2) When received, each burst is coherently processed as usual (Doppler filtering in each range cell);

(3) The outputs of these coherent summations are non-coherently summed (sum of the modulus, or the squared modulus), before final detection thresholding is applied.

This way improves the signal to noise ratio through each coherent burst processing. The resulting non-coherent summation allows consideration of observations at different frequencies, involving different target radar cross sections (RCS). In practice, these different bursts also generally use different repetition frequencies, allowing removal of the ambiguities in range and Doppler [1].

For a high required probability of detection, “some” non-coherent integration is preferable, in order to avoid getting trapped in a low RCS zone, especially for highly fluctuating targets (e.g. Swerling 1 targets, in the standard classification of targets fluctuations [1], [2]). “Some” means that coherent integration must first be used to get a sufficient signal-to-noise ratio (SNR) which should typically be larger than 0dB after coherent integration, so that it is not too much degraded by the modulus operation which, as every nonlinear operation, severely reduces the detection capability if it is done at low SNR. This explains the often used “Golden Rule”: first improve SNR through coherent integration, and then mitigate the low RCS zones by sending a few bursts with frequency agility from burst to burst. The price to pay for that noncoherent integration (and the associated diversity gain) when the available time-on-target is limited, is a lower Doppler resolution because of shorter coherent bursts.

The next question is: how many bursts are required during the time on target? Figure 2 gives a generic answer by showing the required SNR per burst as a function of the number of bursts on the target: $N=6$, or 10 , for Swerling targets [1], [2], [5]. The diversity gain can be defined as the difference in the required SNR per sample, between the coherent summation case and the noncoherent summation case. It clearly appears that the diversity gain is maximum for $N=6$: between 2.6 and 7 dB, depending on the required probability of detection, for this case of Swerling case 1 and 2. There is still a gain for higher resolutions, but it is much smaller.

Similar analyses with Swerling case 3 and 4 targets show that the gain is lower—as expected, since the fluctuations of Swerling 3 targets are smaller than those of Swerling 1—but still exists, at least for detection probabilities larger than 0.8 (i.e., gain between 2.7 and 1 dB). The diversity gain would then become a loss for very high resolution of Swerling 3 targets (1 dB loss for a target analyzed in 30 cells and a required $P_D = 0.8$, not shown in the figure).

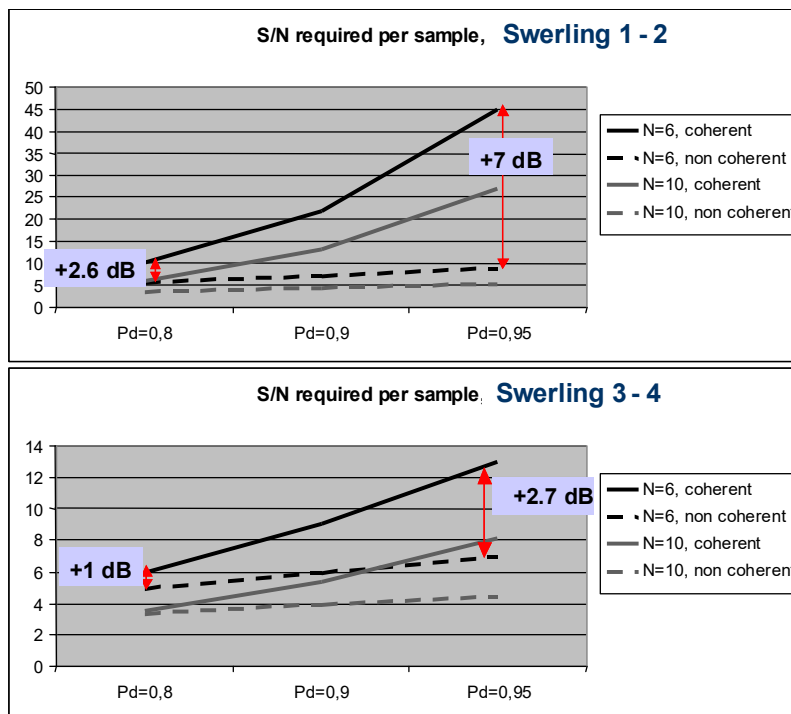


Figure 2: The effect of spatial resolution cells for fluctuating target detection. The traces show noncoherent versus coherent integration for a $P_{fa} = 10^{-6}$.

This basic analysis of diversity leads to the following conclusions:

- The more the target fluctuates, the higher the diversity gain.
- Higher requirements in the probability of detection P_d lead to a higher diversity gain.

- The number of bursts should be between 5 and 10, not more.

A very similar reasoning could apply in the angular / spatial domain, as just shown here in the range / frequency domain. For a multistatic system with a few radar sites, some non-coherent integration will nicely complement coherent integration of the signals received by each site. Depending on the exact signature of the target, spatial diversity and frequency diversity could be preferable: frequency diversity when the scatterers are distributed in range, spatial diversity when the scatterers are distributed in angle. A good solution, if possible, consists in combining both, for example with two or three frequencies per site, and two or three transmitting and/or receiving sites. However, it should be emphasized that frequency diversity is very generally an existing feature on most medium/long range monostatic radars (because most of them use multi-bursts operation, for ambiguity/eclipses removal), whereas spatial diversity, requiring multisite implementations, is only applicable for specific situations, such as passive radars as discussed by Cherniakov in [3] and Chernyak in [4]).

3 Wideband radar detection of moving targets

An essential limitation for standard narrowband radars using bursts of periodic pulses comes from the well-known pulsed radar range-Doppler ambiguity relation, which states that the ambiguous velocity V_a and the ambiguous range D_a are related by $D_a \times V_a = \lambda \times c / 4$. That relation means many ambiguities, either in range or velocity (or both), need consideration. This in turn implies the transmission of successive pulse trains with different repetition frequencies, requiring more time to be spent on target for ambiguity and blind speeds removal (without a corresponding gain in Doppler resolution, since the successive coherent pulse trains are then processed incoherently).

An alternative solution [1] consists in improving the range resolution (through increasing the instantaneous bandwidth) so that the moving target range variation (range walk or range migration) during the pulse train becomes non-negligible compared with the range resolution. Such radars may use bursts with a low pulse repetition frequency (no range ambiguities) and wideband pulses such that the range walk phenomena during the whole burst is significant enough to remove the velocity ambiguity (the range walk being a non-ambiguous measurement of the radial velocity). It then becomes possible to detect the target and measure range and velocity with only one long coherent pulse burst.

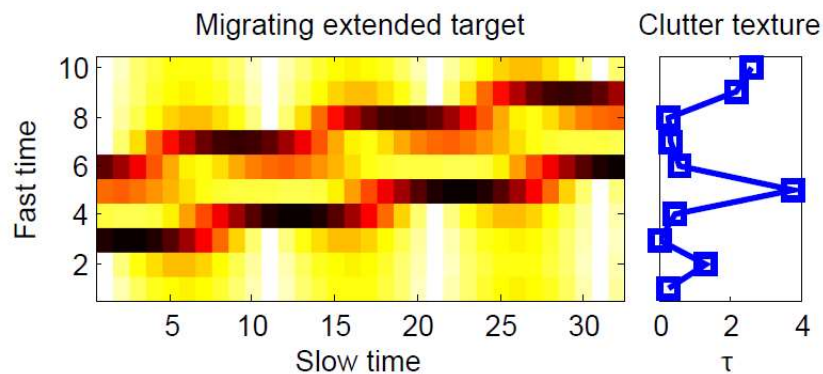


Figure 3. Range migrating extended target in spiky clutter

With such wideband radars, using Parseval's theorem, we first observe that the energy in the squared modulus of the impulse response (range profile) is the same as the energy in the squared modulus of the frequency response. So, summing the energy of the impulse response along the length of the target is equivalent, from a detection point of view, to summing the energy of the corresponding frequency response.

Integration along the range profile of the target for a pre-assumed length of the targets of interest (e.g., 15 m for air targets) is a way to combine coherent integration, used to obtain the range profile with its associated Doppler spectra in each range cell, with noncoherent integration. In other words, for wideband radars, coherent integration time — and the clutter separation that it provides — needs not be reduced to take benefit of diversity gain. Thus summing the bursts in each range cell of a narrowband agile radar is equivalent to summing the samples of the range profile of a high range resolution radar.

An appropriate detector for such wideband radars has been designed [6], ensuring CFAR performance with respect to the clutter texture, speckle correlation matrix, and target velocity. Comparison in Figure 4 includes the CFAR detector DIM-LRT (for Dependent Interference Model, Likelihood Ratio Test), and the clairvoyant detector

(assuming known correlation matrix of clutter). The loss of the proposed detector in comparison to the clairvoyant one is about 1 dB in each scenario. The analysis shows that target detection performance does not depend on clutter spatial correlation γ , but the detection performance depends on target velocity. Thus, the detection gain for the target with velocity $v_0 = 15$ m/s, which obeys a range-walk of about 3 range cells during the CPI, with respect to the stationary one is about 7 dB in K-distributed clutter with shape parameter $\nu = 0.5$.

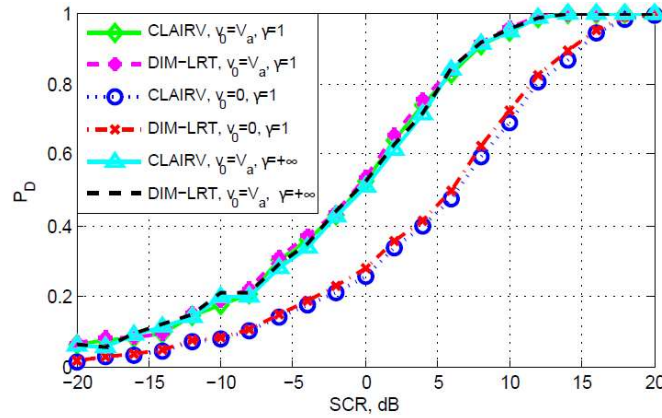


Figure 4. Detection probability of range migrating target in CG clutter with: $v = 0$ m/s and $v = 15$ m/s, SCR is the power of clutter after whitening, exponential correlation in range with $\gamma = 1$ and $\gamma = +\infty$; PFA = 10^{-5} .

The parameters of the radar are: $f_c = 10$ GHz, $B = 1$ GHz, $\delta_R = 0.15$ m, $T_r = 1$ ms, $M = 32$.

This phenomenon – improved detection of fast moving targets – can be well explained by the diversity of clutter, obtained by coherent integration of the target response during its migration over a few range cells. The faster the target, the more it migrates, the lower the probability to miss the target due to a possible clutter spike in one range cell, so the higher the probability of detection: target range-walk along non-Gaussian clutter thus provides a new way to exploit clutter diversity. This behavior is similar to detection of range-extended targets in CG clutter, where the detection performance depends on the target extent (see e.g. [7]). The observed diversity gain is not linear and saturates as the number of the range cells increases: we have observed that the major improvement is obtained by the first 3 range cells migration, and fully saturates for the range-walk over 5 range cells.

Generally speaking, the radar range resolution should be selected such, that the target of interest (given its expected dimensions and radial velocity) is spread over 5-10 range cells as a result of its range extent and its range migration. So meter resolution of the surveillance radar is sufficient for detection of typical air targets with a spatial diversity

4 Wide beam surveillance

Standard digital beamforming provides wide angular sector instantaneous coverage with a wide beam illumination on transmit by transmitting through one relatively small subarray, or through multiple sub-arrays with appropriate phase coefficients for widening the beam. In this technique, also known as “beam spoiling”, the multiple directive beams are simultaneously formed on receive through coherent summations of signals received on different subarrays, in parallel for each aiming direction.

Digital beamforming generally does not essentially change the power budget, compared to standard focused exploration, since the lower gain on transmit (due to wider illumination) is traded against a longer coherent integration time (made possible by the simultaneous observation of different directions). In fact, the main benefit provided by digital beamforming is an improved velocity resolution obtained through this longer integration time, especially useful for target identification purposes, or for detection of slow targets in clutter.

However, the improved velocity resolution of this wide beam exploration comes at a cost: i.e. the non-directive beam on transmit, which induces a poorer rejection of echoes coming from adjacent directions. For ground or surface applications, this means that detection of small targets in presence of strong clutter will become more difficult: the clutter echoes from different directions, which were cancelled not only through the Doppler rejection, but also through the angular separation on transmit and on receive, are now less easily rejected.

Moreover, the use of a wide beam on transmit implies that the angular resolution – and accuracy – is only obtained on receive; the angular resolution is thus poorer – approximately by a factor $\sqrt{2}$ – compared to the standard pencil beam solution.

In order to recover this angular separation on transmit (which was basic to standard focused beam techniques), it is necessary to code the transmitted signals (space-time coding), such that the signals transmitted in the different directions be different – and then become separable on receive as shown in Figure 5.

Past

- Send maximum energy on target, for maximization of the radar power budget
 - Pencil beam, scanning the panorama

Past: Pencil beam, scanning the panorama



Present

- Multiple simultaneous receive beams for faster search performance and more time to analyse the detected objects, providing additional information.
 - Limitation : the same waveform is sent in every direction. This severely limits systems discrimination performance in complex environments

Present: Widening on transmit, multiple receivers



Future

- Ubiquitous observation: simultaneously, different signals are sent in different directions, with reception on multiple parallel channels.
 - Combines wide coverage with selectivity on transmit

Future: multiple transmitters, multiple receivers



Figure 5: Pencil beam, Widebeam digital beamforming, Space-time coding (from [3]).

This method uses different signals simultaneously transmitted in different directions, thus jointly coding space and time, and then coherently processed in parallel on receive. Such concepts, first proposed and demonstrated by Drabowitch and Dorey [8,9], should now be considered as mature techniques to be implemented in operational systems. Basically, the main advantages to be gained are a better extraction of targets — especially slow targets — from clutter, multipath, and noise, and a better identification of targets obtained through longer observation times, and possibly wider bandwidths.

The generic configuration is shown on Figure 6: different codes (preferably with constant amplitude, for better efficiency of the amplifiers) are transmitted through the different antenna elements, or sub-arrays (e.g. lines, or columns of a 2D array); The resulting signal is transmitted by the antenna, resulting in different modulated signals in the different directions.

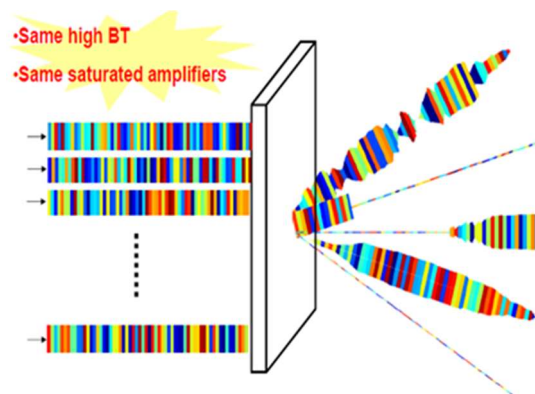


Figure 6: Space-time coding, generic case (from [5])(courtesy J.P. Guyvarch).

The main properties of space-time coding – as far as we are concerned here – are summarized in their range-angle ambiguity function. Indeed, due to the fact that different signals are transmitted through the different antenna elements, the result is a coupling between the range and angle information, so the angular diagrams cannot be analyzed without looking simultaneously at the range domain (the Doppler domain is not affected, since these coding are inside each pulse, and repetitive from pulse to pulse: the Doppler selectivity and Doppler sidelobes are then just as usual).

Let's consider these ambiguity function for "Circulating Codes", which are a good example of such space-time coding, with nice properties as detailed in [5] and [10]. Circulating Codes are generated, as shown on Figure 7, by

the same waveform (e.g. a chirp), “circulating” with a relative time shift δt through N transmitter channels. The relative time shift δt between adjacent circulating signals is equal to 1-time sample, $\Delta t = 1/\Delta F$, where ΔF is the signal bandwidth. As detailed in [5] and [11], it is possible to get different properties with space-time codes based on those circulating codes (or analogously on Frequency Diverse Arrays, which are very similar when the circulating signal is a chirp, since a time shift is then equivalent to a frequency shift).

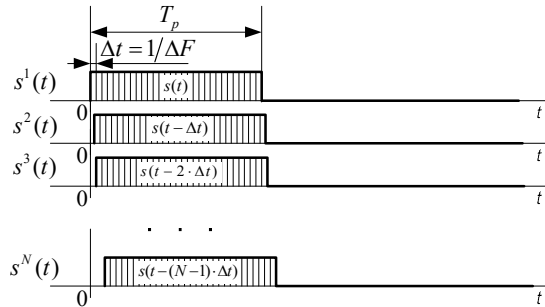


Figure 7: Circulating codes for Space-time coding (from [10]).

$s^i(t) = s[t - (i-1)\Delta t]$ is the signal sent through the i^{th} transmitting element.

For this analysis, the following characteristics are chosen:

Tx/Rx Array: 15 elements, spacing $\lambda/2$; Tx: Wide beam, or space-time coding; Rx: Taylor weighting, 30dB
 Signal: Circulating chirps, pulse duration $t=100\mu\text{s}$, $BT=256$, time delay between adjacent chirps: $\Delta t = 1/B = 0.34\mu\text{s}$; Time weighting (Hamming) on receive. Space-time processing: mismatched « optimal » filtering for Delft codes [12].

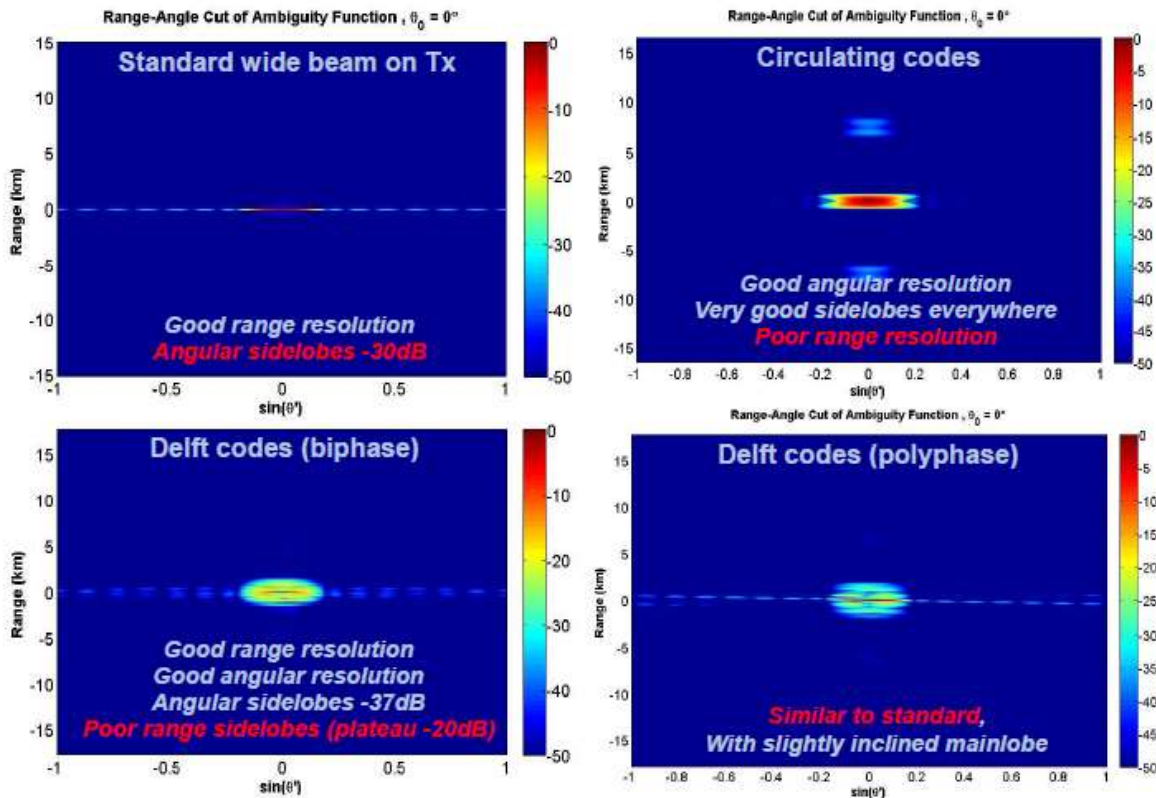


Figure 8: Ambiguity functions for Space-time coding with circulating codes and their variations (from [3]; Delft codes are circulating codes combined with a fixed phase code applied along the antenna array).

The main results can be outlined by looking at the range-angle ambiguity functions (output of the processing for every range and angle, when one target is present at range 0 – arbitrary value – and angle 0), shown on Figure 8:

- The angular resolution is improved with space-time coding, by a factor $\sqrt{2}$, (or 2 for a bidimensional coding of a rectangular array); this is not obvious on the figure, but careful analysis indeed gives this

result (a natural result, actually, since there is now directivity both on transmit and receive, compared with the directivity on receive only for the standard digital beam forming with wide beam on transmit);

- The position and level of the sidelobes are different with the different codings: in angle only for standard beam forming, almost no sidelobes but a degradation in range resolution for pure circulating codes, at different levels and positions for Delft codes [5].

As a result, the increased degrees of freedom provided by space-time coding on transmit open the way to adaptive systems where range and angle resolutions can be traded, depending on the mission and the actual environment (knowledge aided system). This trade-off can be operated differently for each burst, allowing some diversity on clutter to be obtained when several bursts are used for ambiguity removal, for instance. And of course, space-time coding also provides the improvement (mentioned above) in both accuracy and resolution higher than 2, for 2-dimensional antennas, compared with modern wide beam DBF Systems.

5 Discussion

A question then arises: how to combine diversity effects when using agile waveforms? Let us take a baseline example, with a typical modern radar using digital beamforming in elevation only, and a chirp waveform with pulse length 100 μ s, pulse repetition frequency 1 kHz. Any designer would like to benefit from:

1. High Doppler resolution, for visibility of slow and weak targets;
2. High angular resolution, in elevation (for altitude measurement) and azimuth (for tracking);
3. Diversity on the target, for improved detection in noise;
4. Diversity on clutter, for improved detection in clutter.

The first point requires long coherent integration time – but anyway this coherent integration time is limited by the fluctuations of the aspect angle of the target, typically to less than 100ms⁽²⁾.

The second point requires accurate angular measurements (monopulse) with narrow beams on transmit and receive.

The third point requires observations at different carrier frequencies, or from different aspect angles (multistatic system), or integration along a high resolution range profile.

The fourth point requires the target to be superposed to different patches of clutter, either through range migration or range extent of the target, or through multi-bursts with different range ambiguities (so that the target folds over different clutter patches).

These requirements tend to eliminate standard solutions, such as pencil beam with low range resolution (limited velocity resolution due to a short time on target), or standard digital beam forming with no ambiguity in range (limited angular resolution due to the wide beam, limited diversity on clutter)

Facing those trade-offs, several baseline solutions can be sketched:

- a. Pencil beam, high range resolution, unambiguous in range (low PRF): this satisfies all requirements if the coherent integration time is sufficient – and also provides valuable target analysis capabilities, with high resolution range-Doppler signatures.
- b. Space-time coding, low range resolution, ambiguous in range (high/medium PRF): this also satisfies all requirements; pure circulating codes could be a preferred solution in strong clutter environments, providing very low sidelobes everywhere.
- c. Space-time coding with high range resolution, unambiguous in range (low PRF): low sidelobes, high diversity, combined with valuable target analysis capabilities, with high resolution range-Doppler signatures.

These baseline descriptions should of course not be considered as definitive solutions to the very complex task of defining a multifunction radar (for instance, multistatic solutions could also make sense, possibly combined with space-time coding for solving the “rendez-vous” issue, cf [4]). The objective was rather, as outlined in introduction, to highlight and clarify some specificities of diversity effects which have to be considered when designing future systems. Many other aspects, from complexity and cost to multifunction requirements, have also to be taken into consideration – and should also bring out different advantages of high resolution and space-time coding for surveillance radars.

² There may also exist rapid fluctuations of the target due to moving parts (jet engines, rotating blades, wheels) ; however, for moving targets detection, these parts have lower radar cross-sections than the cell of the target, so they can be ignored in this general assessment.

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